A Compressible Turbulent Flow in a Molecular Kinetic Gas Model

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Abstract. This is the continuation of the effort to utilize a kinetic molecular model approach to the problem of turbulence oriented computation of compressible flow. The Boltzmann equation is employed in its integral form with the BGK model of the collision term. This time, we compute a periodic compressible flow inside a unit cube from a random velocity field having an energy spectrum of isotropic type, uniform density and temperature. Results show the development of many small shock waves like structures along with vortex or eddy shocklets scattered in the entire flow field. While the energy spectrum does not change much in its pattern with time, as should be the case, some quantities like density distribution changes quickly to a turbulent state from its initial uniform one. Some geometric properties of computed flow field are derived from its velocity deformation tensor.

INTRODUCTION

We have been concerned with the kinetic molecular model approach to the problem of turbulence oriented computation of compressible flow and considered the Taylor-Green type initial value problem [1],[2]. This was to see the phenomenon of the production of small eddies from large ones, whose mechanism was considered to be one of the basic features of turbulent flow. Here we consider the feature of developed turbulent field. There have been some attempts to this problem with use of lattice Boltzmann equation [3]. Also more recently an interesting preliminary study for this problem with DSMC approach is reported [4]. We compute a periodic compressible flow inside a cube started from a random velocity field with an energy spectrum having the nature of an isotropic turbulence. We use, as in the previous investigations the Boltzmann equation in its integral form, which is reduced for small time step Δt as

$$f(\boldsymbol{c}, \boldsymbol{x} + \boldsymbol{c}\Delta t, t + \Delta t) - f(\boldsymbol{c}, \boldsymbol{x}, t) = \Delta t \, \frac{\partial_e f}{\partial t}, \tag{1}$$

where $f = f(\boldsymbol{c}, \boldsymbol{x}, t)$ is the molecular velocity distribution function with the molecular velocity $\boldsymbol{c}$ and the spatial coordinate $\boldsymbol{x}$, and $\partial_e f/\partial t$ denotes the rate of change in molecular particle number owing to their encounters. In practice, we utilize BGK model for the term $\partial_e f/\partial t$, and evaluate the value at $\boldsymbol{x} - \boldsymbol{c}\Delta t$, which is not necessarily a mesh point with use of a linear interpolation from triangle (for 2D case) or tetrahedron (for 3D case) of neighboring mesh points. The consistency of the present method with the other approaches such as with a different collision model or the Navier-Stokes CFD method has been observed well in the previous investigation [1]. Note also both the similarity and the difference between eq.(1) and the lattice Boltzmann equation [3] that is a discrete model.

ISOTROPIC TURBULENT FIELD

Consider here an initial value problem to an isotropic turbulent field that is periodic to a cube region. We set for this a random initial condition at the time given by a local Maxwellian with its flow velocity $\boldsymbol{u}$, density

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14. ABSTRACT

This is the continuation of the effort to utilize a kinetic molecular model approach to the problem of turbulence oriented computation of compressible flow. The Boltzmann equation is employed in its integral form with the BGK model of the collision term. This time, we compute a periodic compressible flow inside a unit cube from a random velocity field having an energy spectrum of isotropic type, uniform density and temperature. Results show the development of many small shock waves like structures along with vortex or eddy shocklets scattered in the entire flow field. While the energy spectrum does not change much in its pattern with time, as should be the case, some quantities like density distribution changes quickly to a turbulent state from its initial uniform one. Some geometric properties of computed flow field are derived from its velocity deformation tensor.

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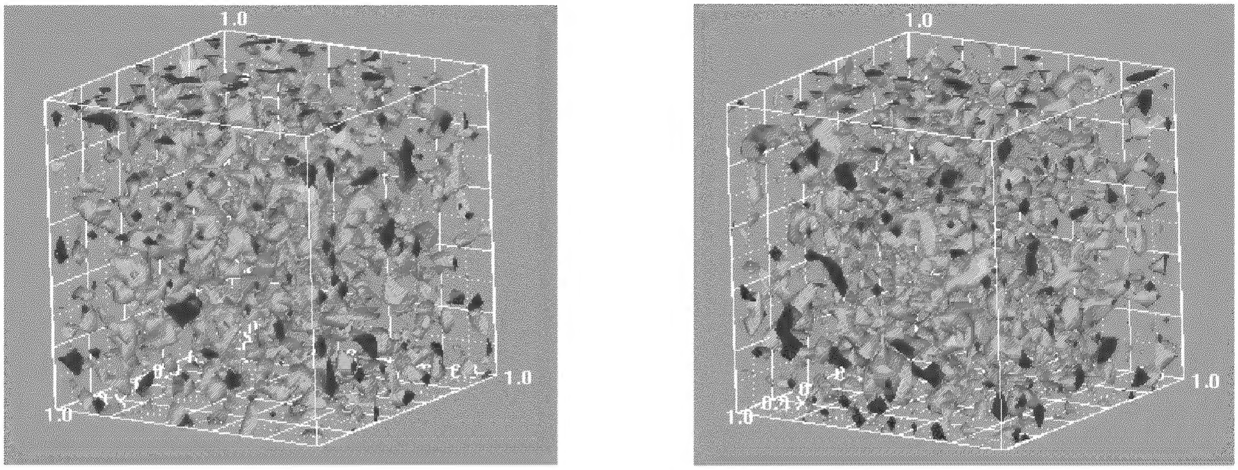


(a) Density (b) Vorticity

FIGURE 1. Iso-surface of density (a) and vorticity (b) distributions at t=0.0125 for K_n=0.05.

ρ and temperature T for $\boldsymbol{x} = (x, y, z)$ where the magnitude of $\boldsymbol{u}$ is given by the energy spectrum $E(\boldsymbol{k})$ of isotropic nature as

$$E(\boldsymbol{k}), E = K^2 k^4 exp\{-2(k/k_0)^2\}, \quad k_0 = 10, \ k = |\boldsymbol{k}| \text{ for wave vector } \boldsymbol{k}, \tag{2}$$

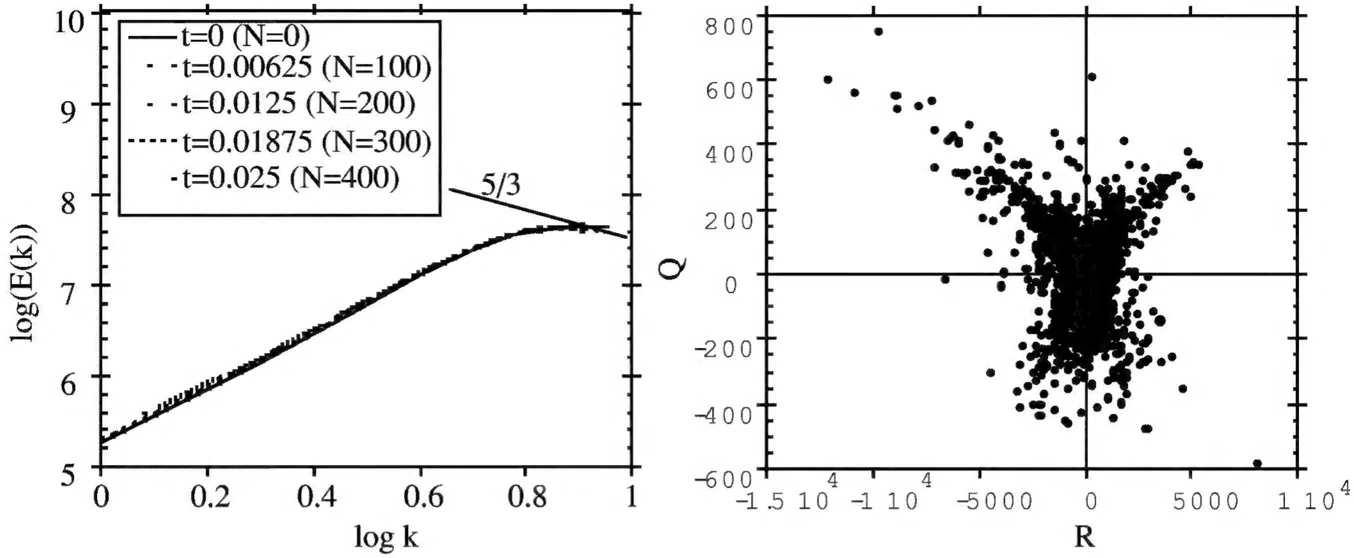
while its phase is givenin random, and ρ, T are set uniform for simplicitys sake. Here all quantities are dimensionless based on the length of an edge of the cube, the initial uniform density and temperature. We use Mach number M =4.3 (K = 0.07) to this initial field given by the velocity to give the maximum energy in $\boldsymbol{k}$-space. We have a relation between the Reynolds number R_e, the Knudsen number K_n and the Mach number M as $R_e K_n = SM$, ($S = 8/5\sqrt{\pi} \simeq 0.9$ for for a Maxwell molecular model).

RESULTS

Computational range of $\boldsymbol{x}$ is a cube region. $\boldsymbol{c}$-space is, as usually been practiced, restricted to a finite region $|\boldsymbol{c}| \leq 4$ by setting outside. We set 25×25×25 divisions for $\boldsymbol{x}$ and 8×8×8 divisions for $\boldsymbol{c}$. Results give distributions of ρ, p, T and $\boldsymbol{u}$ over $\boldsymbol{c}$ at various time levels. These are also utilized to derive some quantities characteristic to the flow field such as the energy spectrum $E(\boldsymbol{k})$, and invariant of the deformation tensor of velocity field. Some representative examples from these results are shown in the following figures. Fig.1 shows iso-surfaces of density (a) and vorticity (b) distributions at t=0.0125 for K_n=0.05 so that the initial Reynolds number R_e=128 for M =4.3. We can see the density distribution changes quickly to a turbulent state from its initial uniform one. We see both in (a), (b) that the development of many small shock waves likes structures combined with vortices and eddy- shocklets [5] scattered in the entire flow field. Fig.2(a) shows the energy spectra at various time levels for K_n=0.05. The pattern do not change much with time, as it should be to the case, and they all show the tendency of deviating of the slop of curve from -5/3 in contrast to the Taylor-Green type flow case [2]. Some geometric properties of flow field [6] are studied by examine the velocity deformation tensor $E = \{e_{ij}\}$, where $e_{ij}$is its components. The values of its invariant P, Q and R defined as

$$P = -\det(e_j), \quad Q = e_{11}e_{22} + e_{22}e_{33} + e_{33}e_{11} - e_{23}e_{32} - e_{12}e_{21} - e_{13}e_{31} = \mathrm{div}\boldsymbol{v} \tag{3}$$

are computed for every mesh points of $\boldsymbol{x}$ at some instants t and they are plotted in planes (P, Q), (Q, R) and (P, R). Of these (Q, R)-plots at t=0.0125 are shown in Fig.2(b). We can notice there a tendency of these points concentrating near the line $Q = -R$ indicating the characteristics of the flow field in compressible turbulence.



(a) (b)

FIGURE 2. (a) Energy spectra and (b) spatial distribution of invariant:(Q, R)-plot of velocity deformation tensor at t=0.0125K_N=0.05 for K_n=0.05.

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